Interest Parity Conditions

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Overview of conditions

- covered interest parity: Investors are covered against nominal uncertainty because of the forward market..
- uncovered interest parity: Equalization of expected nominal returns across borders (no compensation needed).
  - perfect capital mobility
  - perfect capital substitutability
- Real interest parity: Equalization of returns in purchasing power terms.
  - relates to price convergence
  - implies equalization of marginal product of capital

Notation and Concepts

variable	name	definition
S_t	spot exchange rate	the current price of foreign currency. foreign dollars are hence worth s_t euros, $\$ = S_t$ €. Also, $ln(S_t) = s_t$.
$F_{t,t*1}$	forward exchange rate	the price in time t for currency delivered in time $t+1$
i	the domestic interest rate	one € invested at the start of period t will be worth $(1+i)$€ at the end of the period
i^*	the foreign interest rate	one \$ invested at the start of period t will be worth $(1+i)$\$at the end of the period
${S^e}_{t,t+1}$	the expected future spot rate	one \$ note the use of e as an expectations operator.

NOTE 1: some of your readings will be using different notations. For example, the expecations operator may be $E[.]$, and the exchange rate might be ϵ.
NOTE 2: We assume we can move assets between dollars and euros, and can enter and exit forward contracts for currencies and can we can also buy and sell currencies on the spot market.

We can re-arrange this equation. With some manipulation we can arrive at the following:

$$\frac{(i_t - i_t^*)}{(1 + i_t^*)} = \frac{F_{t,t+1} - S_t}{S_t} \tag{1}$$

This is the covered interest parity condition. It states that, as long as investors can cover their risk with a set of forward contracts, they will be able to arbitrage interest rates so that the nominal cross-currency rates of return are equalized. A similar relationship would also then hold for U.S. investors looking at € denominated assets.

$$(1 + i_t^*) = (1 + i_t) \frac{S_t}{F_{t,t+1}} \tag{2}$$

Consider the domestic and foreign rates of return:

$$(1 + i_t) \quad vs \quad (1 + i_t^*) \frac{F_{t,t+1}}{S_t} \tag{3}$$

We have assume here that we have to pay price S_t to buy foreign currency (U.S. dollars) but we can then use this money to buy a dollar-denominated asset that pays i^*. At then end of the (so far vague and undefined) period we can then trade these dollars back into euro through a forward contract, at the rate $F_{t,t+1}$. Assuming a full set of markets, and ignoring transaction costs, the opportunity for arbitrage implies that these two rates will actually be equalized.

$$(1 + i_t) = (1 + i_t^*) \frac{F_{t,t+1}}{S_t} \tag{4}$$

Next, assume that the forward rate is equal to the expected future spot rate:

$$S^{e}{}_{t,t+1} = F_{t,t+1} \quad (5)$$

This allows us to rewrite the covered parity condition in terms of expectations. In particular, this yields the uncovered interest parity condition, wherein expected returns are equalized (even without forward contracts being made).

$$\frac{(i_t - i_t^*)}{(1 + i_t^*)} = \frac{S^e_{t,t+1} - S_t}{S_t} \quad (6)$$

This would follow from forward looking expectations, and markets that are efficient in using information on expectations. Risk-neutra investors will not require compensation for cross-currency operations if this condition holds.

The uncovered interest parity conditions are sometimes expressed in logs. This can be useful, for example, when we want to test the condition using exchange rate and interest rate data. As long as interest rates are low enough, we can rewrite the uncovered interest parity condition as follows:

$$(i_t - i_t^*) = f_{t,t+1} - s_t \tag{7}$$

This has been described (see Frankel 1991, Chen 2007) as a perfect capital mobility condition. Obviously, the condition follows from arbitrage across markets, hence it follows from sufficient (financial) capital mobility. There is a related condition, in logs, that follows from perfect asset substitutability.

$$(i_t - i_t^*) = s_{t,t+1}^e - s_t \tag{8}$$

This condition, another version of the uncovered interest parity condition, says that forward and expected spot rates will simply reflect interest differentials – as long as assets are viewed as substitutes. If U.S. and ECB bonds receive different risk grades from investors, this last condition may then break down.

We might also expect investors to care about real return, where there is also risk of inflation. This implies that real returns, adjusting for changes in price levels, should be equalized. This is real interest parity.

$$i_t - \left(p^e_{t,t+1} - p_t\right) = i^*_t - \left(p^{*e}_{t,t+1} - p^*_t\right) \qquad (9)$$

Again, this is a very special condition in practice. It follows (as we will discuss this term) from full capital mobility. It also implies, in a world where goods prices are equalized, that returns to capital are also equalized across countries. Consider, for example, that if inflation is roughly comparable for a core set of euro countries, this condition says that real returns to capital ought to be equalized.

Basic arbitrage conditions

name	definition
covered interest parity	$\frac{(i_t - i_t^*)}{\left(1+i_t^*\right)} = \frac{F_{t,t+1} - S_t}{S_t}$
uncovered interest parity	$\frac{(i_t - i_t^*)}{\left(1+i_t^*\right)} = \frac{S_{t,t+1}^e - S_t}{S_t}$
real interest parity	$i_t - \left(p_{t,t+1}^e - p_t\right) = i_t^* - \left(p_{t,t+1}^{*e} - p_t^*\right)$

In theory, we can also write these in terms of U.S. foreign investors, taking the exchange rate $1/S_t$ and deriving similar results linked to the forward rate $1/F_t$. When we have asymmetric views (premiums for) risk, or there are transaction costs, these relationships will hold subject to inequality constraints. (With transactions costs, for example, the nominal interest spread give forward markets is bounded by the transaction costs. It cannot be too profitable to arbitrage, given costs.)

The forward premium is the difference between the spot and forward exchange rate today, and should in theory reflect expectations (i.e. information) regarding factors driving future spot rates. We can derive this formally by combining the covered and uncovered interest parity conditions.

$$f_{t,t+1} - s_t = s^e_{t,t+1} - s \tag{10}$$

Here, $f_{t,t+1} - s_t$ is the (log) forward premium. It is an unbiased predictor of the expected depreciation of the domestic currency. As such, it will feature when we discuss econometric evidence on exchange rates. Two issues are embodied in the log forward premium. The first is the information embodied in the expectations term $s^e_{t,t+1}$. This relates to exchange market efficiency. The second involves risk neutrality. If investors are risk averse, then we may observe a risk premium or spread.

- The Efficient Market Hypothesis (EMH) holds that exchange rates (the spot and forward rates) incorporate all relevant information currently available.
- Indeed, under the EMH the forward premium should correctly anticipate subsequent changes in the exchange rate. This is because both the spot and forward rates include the same information set. Formally, it means that the prediction error

$$\epsilon_t = f_{t,t+1} - s_t - \left(s^e_{t,t+1} - s_t\right) \qquad (11)$$

is not systematically related to observable information available at the same time the rates are set (i.e. in time t.) The econometric literature tests this hypothesis by regressing changes in spot rates against the forward-spot rate spread in time t.

$$s_{t,t+1} - s_t = a_0 + a_1\left(f_{t,t+1} - s_t\right) + \epsilon_t \qquad (12)$$

-Here the EMH means that $a_0 = 0$ and $a_1 = 1$.

- If we have a risk premium, meaning that investors are not risk neutral but rather risk-averse, then the interest parity condition needs to be rewritten. Starting in levels:

$$(1 + i_t) = (1 + i_t^*) \frac{F_{t,t+1}}{S_t} \rho_t \tag{13}$$

Taking logs (for sufficiently low values of i and i^*), we then have

$$(i_t - i_t^*) = s_{t,t+1}^e - s_t + \ln(\rho_t) \tag{14}$$

Finally, returning to the uncovered interest (risk free) parity condition, we then have, after substitution:

$$\ln(\rho_t) = f_{t,t+1} - s_{t,t+1}^e \tag{15}$$

This definition of the risk premium then requires a modified version of the EMH test equation.

$$s_{t,t+1} - s_t = a_0 + a_1 (f_{t,t+1} - s_t) - \ln(\rho_t) + \epsilon_t \tag{16}$$

Risk Premia in the 1990s